Fractional matching preclusion

———2018年广东省组合图论会议

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1.Matching preclusion

- A matching of G: A function $f: E(G) \rightarrow \{0,1\}$ such that for each vertex v, $\sum_{e \sim v} f(e) \leq 1$ where the sum is taken over all edges e incident with v, denoted by $e \sim v$.

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- A perfect matching of G: A matching satisfying that $\sum_{e \sim v} f(e) = 1$ for every vertex v of G.
- An almost-perfect matching of G: A matching satisfying that there is exactly one vertex v' such that $\sum_{e \sim v'} f(e) = 0$.

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- Matching preclusion set: A set F of edges of G is a matching preclusion set(MP set) if $G - F$ has neither perfect matchings nor almost-perfect matchings.

4.参考文献

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$$\min 1^T y$$

$$\text{s.t. } y^T q^M \geq 1 \ \text{ for every } M \in \mathcal{M} \qquad (1.1)$$

$$y_e \in \{0, 1\} \ \text{ for every } e \in E(G) \qquad (1.2)$$

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1.研究背景

The concept of matching preclusion was introduced by Brigham[1]. As a measure of robustness in the event of edge failure in interconnection networks, as well as a theoretical connection to conditional connectivity, "changing and unchanging of invariants" and extremal graph theory. An interconnection network with a larger matching preclusion number may be considered as more robust in the event of link failures. The following results can be found in [1,5].

1.研究背景

Theorem 1[1]. Let $n \geq 2$ be an integer. Then

$$mp(K_n) = \begin{cases} n-1 & n \quad is \quad even \\ 2n-3 & otherwise \end{cases}$$

Moreover, if $n \geq 6$ is even, then every optimal matching preclusion set is trivial. If $n \geq 11$ is odd, then every optimal matching preclusion set F is trivial and $G - F$ has two isolated vertices.

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Theorem 2[5]. Let $n \geq 4$ be an even integer. Then

$$mp_1(K_n) = \begin{cases} \frac{(n^2+2n)}{8} & \text{if} n \in \{4,6,8\}, \\ 2n-5, & \text{if} n \geq 10 \textit{ and } n \textit{ is even}. \end{cases}$$

Moreover, if $n \geq 12$, then every optimal conditional matching preclusion set F satisfies that $G - F$ has 2 leaves adjacent to the same vertex.

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Theorem 2[5]. Let $n \geq 5$ be odd integer. Then

$$mp_1(K_n) = \begin{cases} \frac{(n^2+4n+3)}{8}, & \text{if} n \in \{5, 7, 9, 11, 13\}, \\ 3n-9, & \text{if} n \geq 15 \textit{ and } n \textit{ is odd}. \end{cases}$$

Moreover, if $n \geq 17$, then every optimal matching preclusion set F satisfies that $G - F$ has 3 leaves adjacent to the same vertex.

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  对于任意$n \geq 8$ and$k \geq 3$ with $n - k \geq 5$

  $$smp(S_{(n,k)}) = n - 1.$$

  Moreover,every optimal strong matching preclusion set is trivial.

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- 在文献[8]中Ruizhi Lin 和Heping Zhang 修改了MP 模型, 得到下列FMP模型:

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$$\text{s.t. } y^T q^M ge1 \quad \text{for every } M \in \mathcal{M} \qquad (1.1)$$

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  The optimal objective value of FMP is called the fractional matching precluison number of G, denoted by $mp_f(G)$. For any bipartite graph G, they give a formula for $mp_f(G)$ and study the relation between $mp_f(G)$ and $mp(G)$. Moreover, they give a lower bound for $mp_f(G)$ of Cartesian product of two bipartite graphs.

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$$\sum_{e \in E(G)} f(e) = |V(G)|/2.$$

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3.1.完全图的分数（强）匹配排除集和条件分数匹配排除数

- When $|V(G)|$ is even, $mp_1(G) \leq fmp_1(G) \leq 2n-5$. So for a graph G with even vertices, if $mp_1(G) = 2n-5$, then $mp_1(G) = fmp_1(G)$.

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- 参考文献[7]中给出:
- $fmp(K_n) = n - 1$ for every integer $n \geq 7$.
- $fsmp(K_n) = n - 2$ for every integer $n \geq 3$.

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- **定理1** $fmp(K_n) = n - 1$, for every even $n \geq 6$.Moreover,the optimal fractional matching preclusion set is trivial.

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- **定理1** $fmp(K_n) = n - 1$, for every even $n \geq 6$.Moreover,the optimal fractional matching preclusion set is trivial.
- **定理2** $fmp(K_n) = n - 1$, for every odd $n \geq 9$.Moreover,the optimal fractional matching preclusion set is trivial.

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- **定理3** $fsmp(K_n) = n - 2$, for every odd $n \geq 9$.Moreover,the optimal fractional strong matching preclusion set is $X_1 \cup \{e\}$ and $X_2 \cup E_3$ (Let $X_1 \subseteq V(K_n)$ such that$|X_1| = n - 1$,Then $K_n - X = K_3$,$e \in E(K_3)$. Let $X_2 \subseteq V(K_n)$ such that $|X_2| = n - 5$,Then $K_n - X = K_5$, E_3 is the edge set of a complete subgraph on 3 vertices).

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- **定理4** Let $n \geq 5$ be odd integer. Then

$$mp_1(K_n) = \begin{cases} \frac{(n^2-1)}{8}, & \text{if} n \in \{5,7,9,11\}, \\ 2n-5, & \text{if} n \geq 13 \textit{ and n is odd}. \end{cases} \quad (1)$$

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- **定理5** Let $n \geq 5$ be even integer. Then

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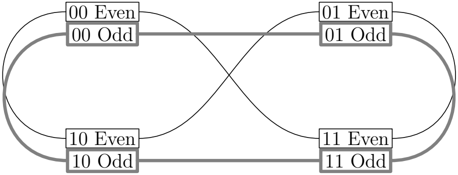
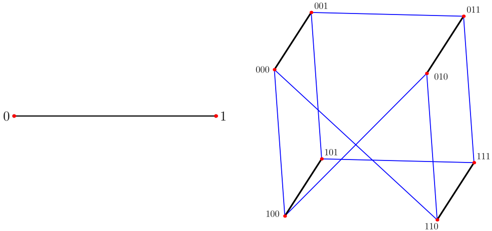


Cross edges

1.符号和定义



twisted 1-cube, twisted 3-cube

3.3.扭曲立方体的分数强匹配排除数

- **定理6** Let $n \geq 5$ be odd. Then $fsmp(TQ_n) = n$ for every odd $n \geq 5$.Moreover,the optimal fractional strong matching preclusion set is trivial. .

3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集

- The (n,k)-star graph,denoted $S_{(n,k)}$ is defined for positive integers n and k such that $n > k \geq 1$. The vertex set of the graph is all the permutations on k elements of the set {1, 2, • • • , n}. Two vertices corresponding to the permutations $[a_1, a_2, \ldots, a_k]$and $[b_1, b_2, \ldots, b_k]$ are adjacent if and only if either:

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- (1) There exists an integer $2 \leq s \leq k$ such that $a_1 = b_s$ and $b_1 = a_s$ and for any $i \neq s, 1 < i \leq k$, we have $a_i = b_i$.(That is, $[b_1, b_2, \ldots, b_k]$is obtained from $[a_1, a_2, \ldots, a_k]$by swapping $a_1$and a_s.)

3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集

- (1) There exists an integer $2 \leq s \leq k$ such that $a_1 = b_s$ and $b_1 = a_s$ and for any $i \neq s$,$1 < i \leq k$, we have $a_i = b_i$.(That is, $[b_1, b_2, \ldots, b_k]$is obtained from $[a_1, a_2, \ldots, a_k]$by swapping $a_1$and a_s.)
- (2)For all $2 \leq i \leq k$, we have $a_i = b_i$ and $a_1 \neq b_1$. (That is, $[b1, b2, \ldots, b_k]$is obtained from $[a_1, a_2, \ldots, a_k]$by replacing$a_1$by an element in $\{1, 2, ..., n\}$-$\{a1, a2, ..., ak\}$.)

3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集

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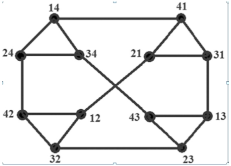
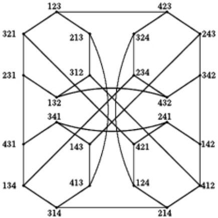
3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集





$S_{(4,2)}$, $S_{(4,3)}$

3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集

- **定理7** $fsmp(S_{(n,2)}) = n - 1$.Moreover every optimal fractional strong matching set is trivial,P_2-trivial.

3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集

- **定理7** $fsmp(S_{(n,2)}) = n - 1$.Moreover every optimal fractional strong matching set is trivial,P_2-trivial.
- **定理8** $fsmp(S_{(n,3)}) = n - 1$.Moreover every optimal fractional strong matching set is trivial.

3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集

- **定理7** $fsmp(S_{(n,2)}) = n - 1$.Moreover every optimal fractional strong matching set is trivial,P_2-trivial.
- **定理8** $fsmp(S_{(n,3)}) = n - 1$.Moreover every optimal fractional strong matching set is trivial.
- **定理9** Let$n > k \geq 3$ with $n - k \geq 2, fsmp(S_{(n,k)}) = n - 1$.Moreover every optimal fractional strong matching set is trivial.

3.4.(n,k)-star graph的分数强匹配排除数和分数强匹配排除集

- **定理7** $fsmp(S_{(n,2)}) = n-1$.Moreover every optimal fractional strong matching set is trivial,P_2-trivial.
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Thanks for your attention!