Strongly and hyper Hamiltonian laceability of balanced hypercubes with faulty edges

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Background

In recent years, the balanced hypercubes have attracted much attention.

Definitions

- path: A path denoted by $\langle v_1, v_2, \ldots, v_n \rangle$ is a sequence of different vertices where two successive vertices are adjacent in G.
- Hamiltonian path: A Hamiltonian path of G is a path containing every vertex of G.
- Hamiltonian connected: If there is a Hamiltonian path between any two vertices of G, then G is Hamiltonian connected.
- Hamiltonian laceable: A bipartite graph G is Hamiltonian laceable if there is a Hamiltonian path between any two vertices from different partite sets.

Definitions

- strongly Hamiltonian laceable A bipartite graph $G = (V_0 \cup V_1, E)$ with $|V_0| = |V_1|$ is strongly Hamiltonian laceable if it is Hamiltonian laceable and there is a path of length $|V_0 \cup V_1| - 2$ between any two distinct vertices from the same partite set [11].
- hyper Hamiltonian laceable A graph G is hyper Hamiltonian laceable if it is Hamiltonian laceable and for any vertex $x \in V_i$ and any two vertices $u, v \in V_{1-i}$, there is a hamiltonian path joining u and v in $G - x$ [16].

[11] S.-Y. Hsieh, G.-H. Chen, C.-N. Ho, Hamiltonian-laceability of star graphs, 485 Networks 36 (4) (2000), pp. 225-232. [16] M. Lewinter, W. Widulski, Hyper-Hamiltonian laceable and caterpillarspannable product graphs, Comput. Math. Appl., 34 (11) (1997), pp. 99- 104.

Definitions

- eftHL(G), eftSHL(G), eftHHL(G): Li et al. [17] used $eftHL(G)$, $eftSHL(G)$, $eftHHL(G)$ to denote edge fault tolerant Hamiltonian laceability, edge fault tolerant strongly Hamiltonian laceability, and edge fault tolerant hyper Hamiltonian laceability of G, respectively.
- eftHL(G), eftSHL(G), eftHHL(G): eftHL(G) (respectively, eftSHL(G), eftHHL(G)) is the minimum integer f such that for any $F \subset E(G)$ with $|F| \leq f$, $G - F$ is still Hamiltonian laceable (respectively, strongly Hamiltonian laceable, hyper Hamiltonian laceable).

T.-K. Li, J.J.M. Tan, L.-H. Hsu, Hyper hamiltonian laceability on edge fault star graph, Inform. Sciences., 165 (2004), pp. 59-71.

Definitions

- *k* edge fault-tolerant Hamiltonian: A faulty graph G with $|F| \leq k$ faulty edges is k edge fault-tolerant Hamiltonian (respectively, Hamiltonian laceable) if $G - F$ remains Hamiltonian.[11]
- *k* edge fault-tolerant strongly Hamiltonian laceable (respectively, hyper Hamiltonian laceable) Defined similarly.

S.-Y. Hsieh, G.-H. Chen, C.-N. Ho, Hamiltonian-laceability of star graphs, 485 Networks 36 (4) (2000), pp. 225-232.

Definitions

Definition (J. Wu, K. Huang, IEEE Trans. Comput., 1997)

$BH_n = (V(BH_n), E(BH_n))$, where $V(BH_n) = \{(a_0, a_1, \ldots, a_{n-1}) \mid a_i \in \{0, 1, 2, 3\}$ for $i \in \{0, 1, 2, \ldots, n-1\}\}$. Each vertex $(a_0, a_1, \ldots, a_{n-1})$ is adjacent to the following $2n$ vertices,
$((a_0 \pm 1) \bmod 4, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1})$, and
$((a_0 \pm 1) \bmod 4, a_1, \ldots, a_{i-1}, (a_i + (-1)^{a_0}) \bmod 4, a_{i+1}, \ldots, a_{n-1})$,
where $1 \leq i \leq n-1$.

J. Wu, K. Huang, The balanced hypercubes: A cube-based system for fault tolerant applications, IEEE Trans. Comput., 46 (4) (1997), pp. 484–490.

Definitions

Definition (J. Wu, K. Huang, IEEE Trans. Comput., 1997)

(1) BH_1 is a 4-cycle denoted by $\langle 0, 1, 2, 3, 0 \rangle$.
(2) For $n \geq 2$, BH_n is constructed by BH^0_{n-1}, BH^1_{n-1}, BH^2_{n-1}, and BH^3_{n-1}. Each vertex $(a_0, a_1, \ldots, a_{n-2}, i)$ in BH^i_{n-1} ($i \in \{0, 1, 2, 3\}$) has two extra neighbors:
(a) $(a_0 \pm 1 \pmod 4, a_1, \ldots, a_{n-2}, i + 1 \pmod 4)$ in BH^{i+1}_{n-1} if a_0 is even,
(b) $(a_0 \pm 1 \pmod 4, a_1, \ldots, a_{n-2}, i - 1 \pmod 4)$ in BH^{i-1}_{n-1} if a_0 is odd.

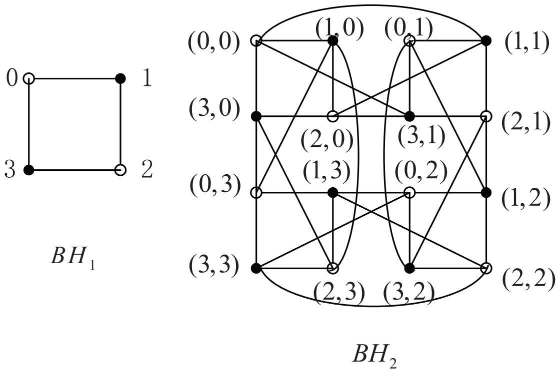

Fig.: The illustrations of BH_1 and BH_2.

Related results

Xu et al. [24] proved that the balanced hypercube is Hamiltonian laceable.

M. Xu, X.-D. Hu, J.-M. Xu, Edge-pancyclicity and Hamiltonian laceability of the balanced hypercubes, Appl. Math. Comput., 189 (2007), pp. 1393- 1401.

Related results

Lü and Zhang [18] further proved that the balanced hypercube is hyper Hamiltonian laceable.

H. Lü, H. Zhang, Hyper-Hamiltonian laceability of balanced hypercubes, J. Supercomput., 68 (1) (2014), pp. 302-314.

Related results

Lemma (Q. Zhou, D. Chen, H. Lü, Inform. Sciences., 2015)

BH_n is $2n-2$ edge fault-tolerant Hamiltonian laceable for $n \geq 2$.

Q. Zhou, D. Chen, H. Lü, Fault-tolerant Hamiltonian laceability of balanced hypercubes, Inform. Sciences., 300 (2015), pp. 20-27.

Useful Lemmas

Lemma (Q. Zhou, D. Chen, H. Lü, Inform. Sciences., 2015)

Let ∂d be the set of edges along dimension d.

Q. Zhou, D. Chen, H. Lü, Fault-tolerant Hamiltonian laceability of balanced hypercubes, Inform. Sciences., 300 (2015), pp. 20-27.

Useful Lemmas

Lemma (Q. Zhou, D. Chen, H. Lü, Inform. Sciences., 2015)

Let $n \geq 2$ be an integer. Then BH_n can be divided into four BH_{n-1}s by deleting ∂d for any $d \in \{0, 1, \ldots, n-1\}$.

Q. Zhou, D. Chen, H. Lü, Fault-tolerant Hamiltonian laceability of balanced hypercubes, Inform. Sciences., 300 (2015), pp. 20–27.

Useful Lemmas

Lemma (J. Wu, K. Huang, IEEE Trans. Comput., 1997)

The balanced hypercube is bipartite and vertex-transitive.

J. Wu, K. Huang, The balanced hypercubes: A cube-based system for fault tolerant applications, IEEE Trans. Comput., 46 (4) (1997), pp. 484–490.

Useful Lemmas

Lemma (J.-X. Zhou, Z.-L. Wu, S.-C. Yang, K.-W. Yuan, IEEE Trans. Comput., 2015)

The balanced hypercube is edge-transitive.

J.-X. Zhou, Z.-L. Wu, S.-C. Yang, K.-W. Yuan, Symmetric property and reliability of balanced hypercube, IEEE Trans. Comput., 64 (3) (2015), pp. 876-881.

Useful results

Lemma (M.-C. Yang, Comput. Math. Appl., 2010)

The balanced hypercube BH_n is bipanconnected for all $n \geq 1$.

M.-C. Yang, Bipanconnectivity of balanced hypercubes, Comput. Math. Appl., 60 (2010), pp. 1859–1867.

Useful results

Lemma (M.-C. Yang, Appl. Math. Comput., 2012)

There exist 2^{2n-2} edges between BH^{i}_{n-1} and BH^{i+1}_{n-1} for each $i \in \{0, 1, 2, 3\}$, where $n \geq 2$.

M.-C. Yang, Super connectivity of balanced hypercubes, Appl. Math. Comput., 219 (3) (2012), pp. 970–975.

Useful Lemmas

Lemma (D. Cheng, R.-X. Hao, Y.-Q. Feng, Appl. Math. Comput., 2014)

Let X and Y be two distinct partite sets of BH_n. Assume that u and x are two different nodes in X, and v and y are two different nodes in Y. Then there exist two node-disjoint paths $P[x, y]$ and $R[u, v]$, and $V(P[x, y]) \cup V(R[u, v]) = V(BH_n)$, where $n \geq 1$.

D. Cheng, R.-X. Hao, Y.-Q. Feng, Two node-disjoint paths in balanced hypercubes, Appl. Math. Comput., 242 (2014), pp. 127–142.

Useful Lemmas

Lemma (T.-K. Li, J.J.M. Tan, L.-H. Hsu, Inform. Sciences., 2004)

Let $G = (V_0 \cup V_1, E)$ be a bipartite graph with $|V_0| = |V_1|$ and let δ be the minimum degree of G among all vertices. We have eftHL(G) $\leq \delta - 2$, eftSHL(G) $\leq \delta - 2$ for $\delta \geq 2$, and eftHHL(G) $\leq \delta - 3$ for $\delta \geq 3$.

T.-K. Li, J.J.M. Tan, L.-H. Hsu, Hyper hamiltonian laceability on edge fault star graph, Inform. Sciences., 165 (2004), pp. 59–71.

Main results

Lemma (D. Cheng, 2018+)

Let BH_n be an n-dimensional balanced hypercube with $|F| = 2n - 2$ faulty edges, where $n \geq 2$. Then BH_n can be divided into four BH_{n-1}s such that (1) if $n \geq 3$, then each sub-balanced hypercube BH_{n-1} contains at most $2n - 4$ faulty edges; and (2) if $n = 2$, then there is at most one faulty edge in some BH_{n-1}.

Main results

Lemma (D. Cheng, 2018+)

BH_2 is 2 edge fault-tolerant strongly Hamiltonian laceable.

Main results

Lemma (D. Cheng, 2018+)

In an n-dimensional balanced hypercube BH_n with at most $2n-2$ faulty edges, there is at least one fault-free edge between adjacent sub-balanced hypercubes, where $n \geq 2$.

Main results

Theorem (D. Cheng, 2018+)

BH_n is $2n-2$ edge fault-tolerant strongly Hamiltonian laceable, where $n \geq 1$.

Main results

Lemma (D. Cheng, 2018+)

BH_2 is 1 edge fault-tolerant hyper Hamiltonian laceability.

Main results

Theorem (D. Cheng, 2018+)

The n-dimensional balanced hypercube BHn is $2n-3$ edge faulttolerant hyper Hamiltonian laceable, where $n \geq 2$.

Thank you for your attention!