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投资者博弈模型及其在复杂网络上的合作演化

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Contents

- Background and Problem
- Review of some fundamental works
- Our works

How Did Cooperative Behavior Evolve?

Elizabeth Pennisi (June 30, 2005)

Science **309 (5731), 93**

合作的意义

Science公布的科学前沿问题：人类合作行为如何演化？

- 合作意味牺牲自己、成全别人，容忍背叛
- 合作是社会发展的基石
- 合作行为是如何在理性的自私个体中演化？

王赟, 魏子晗等. 世纪科学之问“合作+行为是如何进化的”——中国学者的回应. 科学通报, 2016, 61: 20–33

科学通报 2016 年 第 61 卷 第 1 期: 20 ~ 33

《中国科学》杂志社
SCIENCE CHINA PRESS

“Science125个科学前沿问题”系列解读(Ⅰ)

How did cooperative behavior evolve?

世纪科学之问“合作+行为是如何进化的”——中国学者的回应

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摘要 合作+行为普遍存在于动物界和人类社会. 然而, 在激烈的生存斗争中, 合作行为的成因是难以被理解的一个谜题. *Science*在2005年把“合作行为是如何进化的”这一问题列为125个科学前沿问题之一. 本文从中国学者的角度出发, 梳理了国内研究者在合作行为这一领域的研究总体态势及研究现状, 进而从理论模型、行为学、神经基础、分子遗传学等角度探讨了合作行为的机制. 最后总结并讨论了中国学者在合作领域进行研究所拥有的优势及存在的不足, 并对今后的研究进行展望.

为什么产生合作行为?

- 在现有文献中, 尝试对合作或亲社会行为进行解释的潜在机制众多. 国外研究者探索的机制主要包括:
- 信任程度(degree of belief)(即信任或怀疑)、
- 道德系数k(coefficient of morality k)、
- 模仿(mimicry)、
- 亲缘选择(kin selection)、
- 种群选择(group selection)、
- 惩罚(punishment)、
- 顺从(docility)、
- 学习(learning)、
- 互惠(reciprocation)、
- 社会与个人准则(social and personal standards)等.

王赟等.世纪科学之问“合作+行为是如何进化的”——中国学者的回应.科学通报, 2016, 61: 20–33

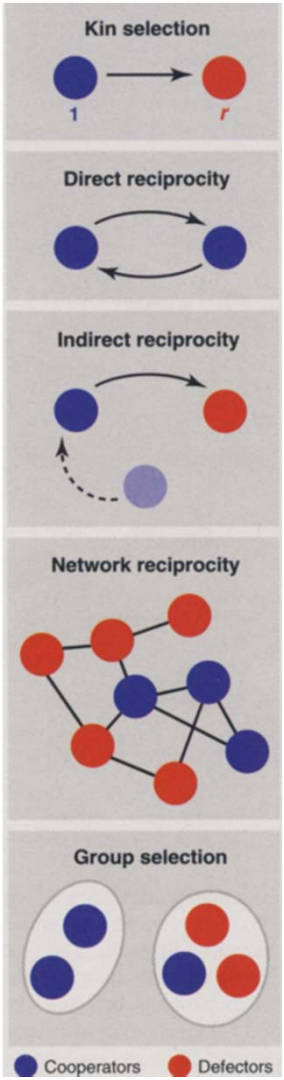
Five mechanisms for cooperation

- Kin selection : cooperate with genetic relatives.
- Direct reciprocity : I help you, you help me.

  (requires repeated encounters between the same two individuals)
- Indirect reciprocity : I help you, somebody helps me.

  (based on reputation; a helpful individual is more likely to receive help)
- **Network reciprocity** : Neighbors help each other

  (Network reciprocity mean that clusters of cooperators outcompete defects)
- Group selection : the idea that competition is not only between individuals but also between groups.



Nowak M A. Five rules for the evolution of cooperation. Science, 2006, 314: 1560–1563

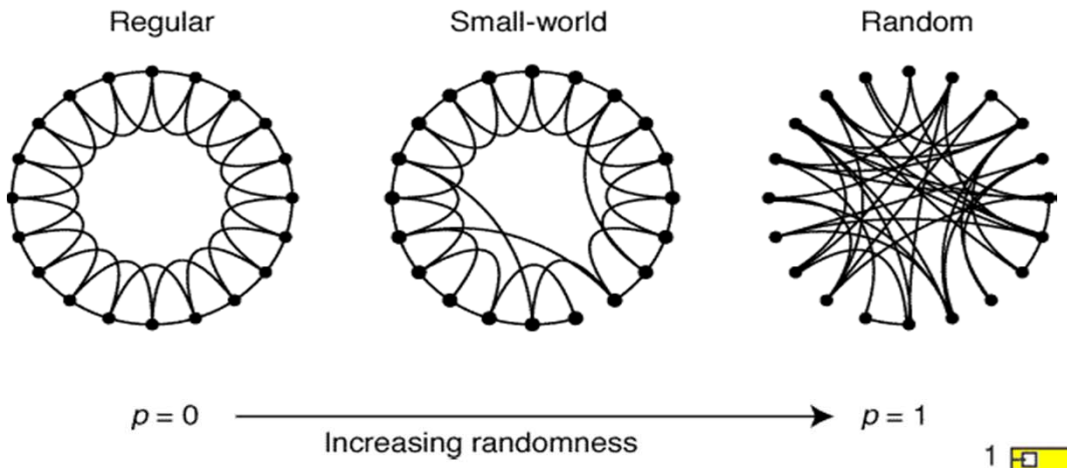
Review of some fundamental works

- Complex Networks
- Evolutionary Games
- Some fundamental works

Complex Networks

- Regular networks
- Random networks
- Small-world networks
- Scale-free networks

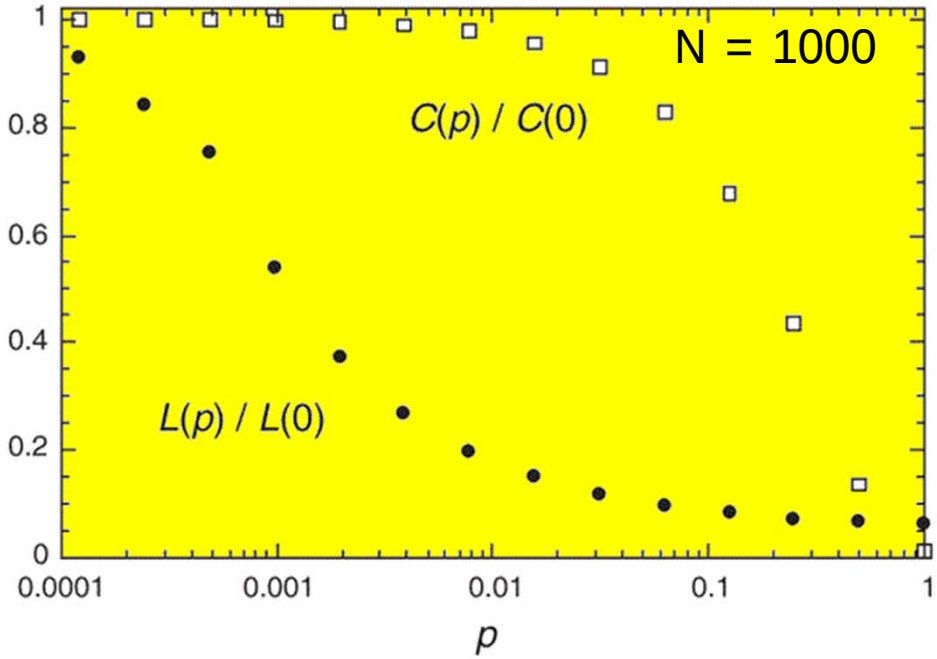
Small-world networks



N nodes forms a regular lattice. With probability p, each edge is rewired randomly

=>Shortcuts

- Large clustering coeff.
- Short typical path



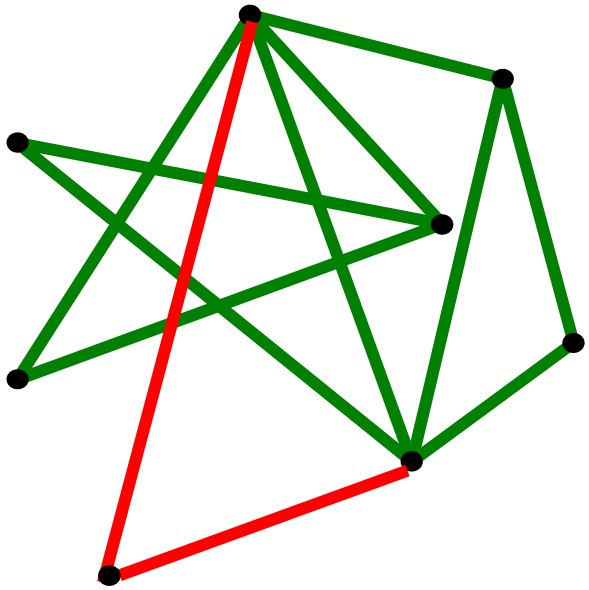
Watts & Strogatz, *Nature* 393, 440 (1998)

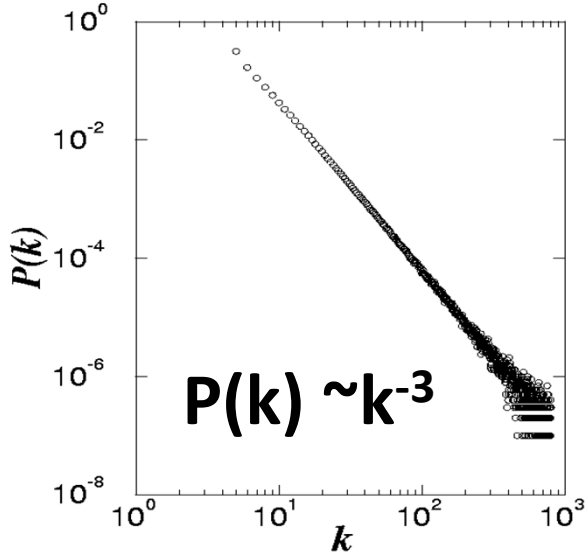
Scale-free model

(1) GROWTH : At every timestep we add a new node with *m* edges (connected to the nodes already present in the system).

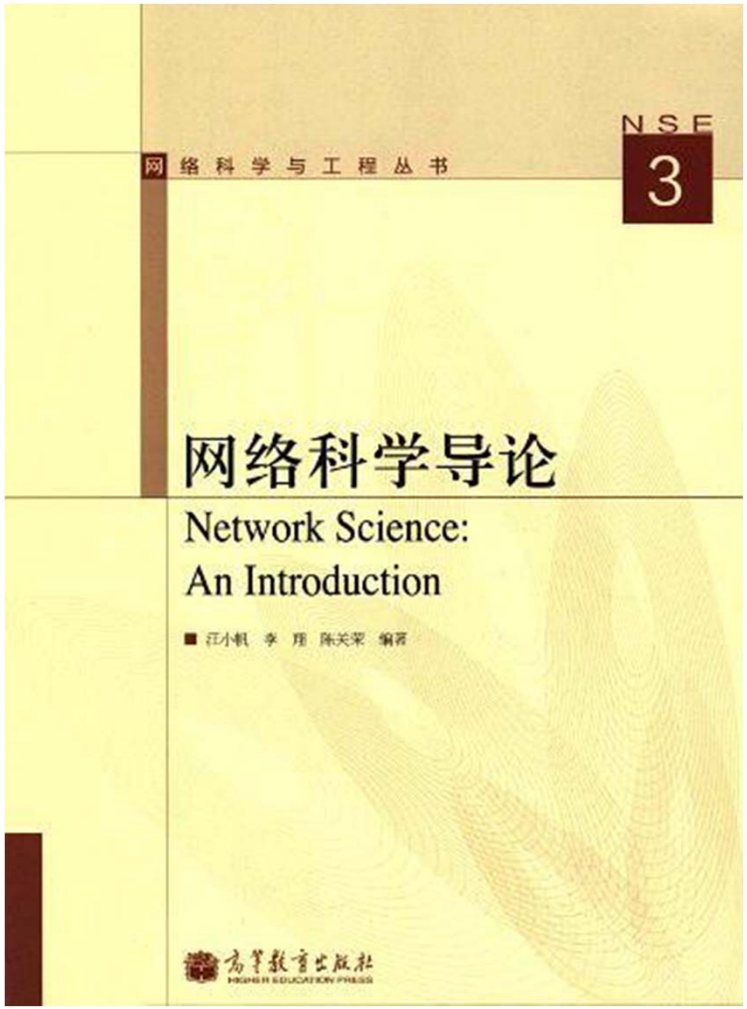
(2) PREFERENTIAL ATTACHMENT :
The probability **Π** that a new node will be connected to node *i* depends on the connectivity *ki* of that node

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$





A.-L.Barabási, R. Albert, Science 286, 509 (1999)



Games

- Prisoner’s dilemma
- Snowdrift games
- Public goods games

Prisoner's Dilemma

- The story of prisoner's dilemma

  Player: two prisoners

  Action: {Cooperation, Defect}

  Payoff matrix

		Prisoner B: C	Prisoner B: D
Prisoner A	C	(3,3)	(0,5)
	D	(5,0)	(1,1)

Prisoner's Dilemma

- No matter what the other does, the best choice is "D".
- (D,D) is a Nash Equilibrium.
- But, if both choose "D", both will do worse than if both select "C"

		Prisoner B: C	Prisoner B: D
Prisoner A	C	(3,3)	(0,5)
	D	(5,0)	(1,1)

Evolutionary Games

- Evolutionary game theory is an extension of the classical paradigm towards bounded rationality.
- There is however, another aspect of the theory which was swept under the rug in the classical approach, but gets special emphasis in the evolutionary version, namely dynamics.
- There is a *static* and a *dynamic* perspective of evolutionary game theory.
- Maynard Smith's definition of the evolutionary stability of a Nash equilibrium is a static concepts which does not require solving time-dependent dynamic equations.

Szabó G, Fath G. Evolutionary games on graphs. Physics reports, 2007, 446(4-6): 97-216.

Evolutionary Games

The mission of evolutionary game theory was to remedy three key deficiencies of the classical theory:

- (1) bounded rationality,
- (2) the lack of dynamics, and
- (3) equilibrium selection in the case of multiple Nash equilibria.

Evolutionary Games

- *Evolutionary game theory* is the theory of dynamic adaptation and learning in (infinitely) repeated games played by boundedly rational agents.
- A *mean-field* or *population game* is defined by the underlying two-player stage game, the set of feasible strategies (usually mixed strategies are not allowed or are strongly restricted), and the heuristic updating mechanism for the individual strategies (update rules).
- *Evolutionarily stable strategies*. The first concept of evolutionary stability was formulated by Maynard Smith and Price (1973) in the context of symmetric population games.
- An *evolutionarily stable strategy* (ESS) is a strategy that, when used by an entire population, is immune against invasion by a minority of mutants playing a different strategy.

Evolutionary Games: Strategy update rules

Describe

- how the agents perceive their surrounding environment,
- what information they acquire,
- what believes and expectations they form from former experience, and
- how this all translates into strategy updates during the game.

Evolutionary Games: Strategy update rules

- Synchronized update
- Random sequential update
- Microscopic update rules

  Mutation and experimentation;

  Imitation(Imitation processes can differ in two respects: whom to imitate and with what probability);

  Moran process;

  Better and best response;

  Win–Stay–Lose–Shift
- From micro to macro dynamics in population games
- Potential games and the kinetic Ising model
- Stochastic stability

Evolutionary Games

The major concern is

- The long run behavior of the system: fixed points, cycles, and their stability, chaos, etc., and
- The connection between static concept (Nash equilibrium, evolutionary stability) , and
- Dynamic predictions.

Szabó G, Fath G. Evolutionary games on graphs. Physics reports, 2007, 446(4-6): 97-216.

10.3 规则网络上的演化博弈

种群的接触关系可以用网络描述——每个节点代表一个个体，节点间的边代表个体之间的相互作用关系，在每一轮中它们根据某个博弈模型进行交互作用，并采取统一的演化规则进行策略的更新。经典的演化博弈理论通常假设个体以均匀混合的方式联系，即任意两个个体之间接触的可能性都是一样的。然而，现实生活中个体之间接触并非是全耦合或者完全随机的。网络结构与演化博弈之间有密切的联系，这方面的研究也称为网络演化博弈(Networked evolutionary game)。博弈模型、网络结构和演化规则是网络演化博弈的3个要素[18]。以下主要围绕两人两策略博弈模型，介绍各种网络结构对博弈行为的影响。

10.3.1 规则网络上的囚徒困境博弈

Nowak 和 May 首先将空间结构引入囚徒困境[19]，研究了二维方格格子上的重复囚徒困境博弈，并考虑了如下定义的较为简单的收益矩阵：

$$\begin{array}{c} \\ \mathrm{C} \\ \mathrm{D} \end{array}\begin{array}{c}\begin{array}{cc}\mathrm{C} & \mathrm{D}\end{array}\\ \begin{pmatrix} R & S \\ T & P \end{pmatrix}\end{array} = \begin{array}{c} \\ \mathrm{C} \\ \mathrm{D} \end{array}\begin{array}{c}\begin{array}{cc}\mathrm{C} & \mathrm{D}\end{array}\\ \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}\end{array} \tag{10-9}$$

其中唯一的可调参数 b 称为“背叛者的诱惑”，当 $b>1$ 时属于囚徒困境情况

Some fundamental works

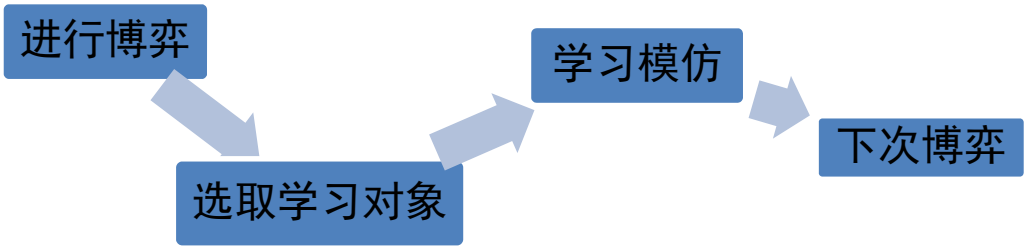
个体和具体的邻居进行重复博弈，通过模仿学习调整自身策略

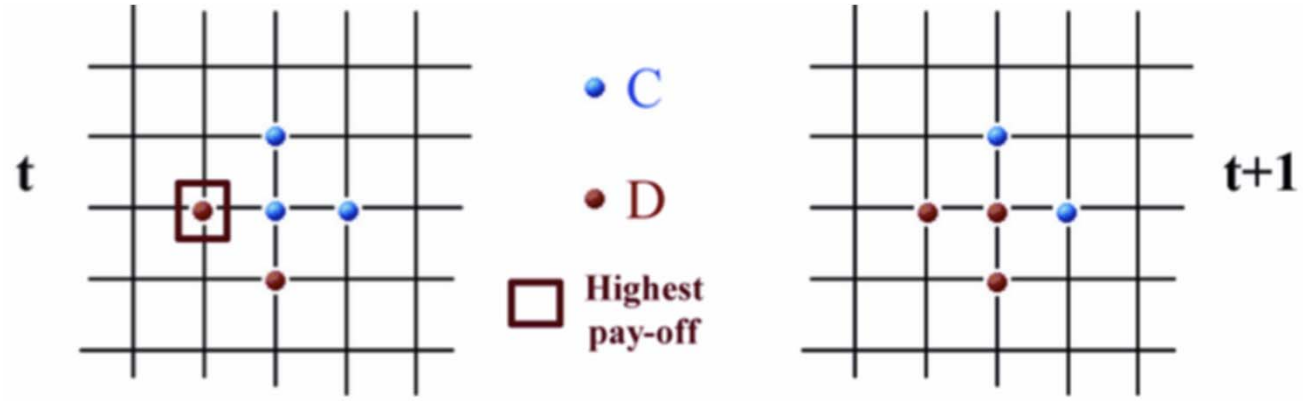
LETTERS TO NATURE

Evolutionary games and spatial chaos

Martin A. Nowak & Robert M. May

Department of Zoology, University of Oxford, South Parks Road, Oxford OX1 3PS, UK





M.A. Nowak, R.M. May, Evolutionary games and spatial chaos, Nature 359 (6398) (1992) 826–829 .

合作者结成团块抵抗背叛者的入侵

Reviews

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- M. Perc, A. Szolnoki, Coevolutionary games--a mini review, Biosystems 99 (2010) 109-125.
- Perc M, Szolnoki A, Floría L M, et al. Evolutionary dynamics of group interactions on structured populations: a review. Journal of the Royal Society Interface, 2013, 10(80):20120997.
- M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, A. Szolnoki, Statistical physics of human cooperation, Phys. Rep. 687 (2017) 1-51

Our works

- 1、Hedong Xu, Cunzhi Tian, Suohai Fan*, and Jiajia Li, Information flows in the market: an evolutionary game approach, Chaos 29, 023126 (2019),Volume 29, Issue 2, February 2019.
- 2、 Yukun Dong, Hedong Xu, Suohai Fan*, Memory-based stag hunt game on regular lattices, Physica A: Statistical Mechanics and its Applications, Volume 519, 1 April 2019, Pages 247-255.
- 3、 Hedong Xu, Suohai Fan*, Cunzhi Tian, Xinrong Xiao. Effect of strategy-assortativity on investor sharing games in the market, Physica A : Statistical Mechanics and its Applications, Volume 514, 15 January 2019, Pages 211–225.
- 4、 Hedong Xu, Suohai Fan*, Cunzhi Tian, Xinrong Xiao. Evolutionary investor sharing game on networks , Applied Mathematics and Computation, Volume 340, 1 January 2019, Pages 138-145.
- **5、Hedong Xu, Cunzhi Tian, Wenxing Ye, Suohai Fan*. Effects of investors' power correlations in the power-based game on networks, Physica A 506 (2018) 424–432.**
- **6、 Hedong Xu, Cunzhi Tian, Xinrong Xiao, Suohai Fan*. Evolutionary investors' power-based game on networks, Applied Mathematics and Computation , Volume 330, 1 August 2018, Pages 125–133**.
- 7、Wenxing Ye, Weiying Feng, Chen Lu, Suohai Fan*. Memory-based prisoner's dilemma game with conditional selection on networks， Applied Mathematics and Computation ， Volume 307, 15 August 2017, Pages 31–37.
- 8、 Wenxing Ye, Suohai Fan*. Evolutionary snowdrift game with rational selection based on radical evaluation, Applied Mathematics and Computation ， Volume 294, 1 February 2017, Pages 310–317.

Applied Mathematics and Computation 330 (2018) 125–133

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Evolutionary investors' power-based game on networks

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Cooperative behavior
Market efficiency
Evolutionary process

ABSTRACT

The classical prisoner dilemma game on networks ignores the heterogeneity of players that may lead to the remarkable differences of their payoffs in reality. With the consideration of the heterogeneity, we propose an investors' power-based game, where the payoffs of defectors depend on the efficiency of market and the related-power against cooperators. Economically, the efficiency of the market of investment is introduced in the game through a parameter α that becomes a key factor in the evolutionary process. Our simulation results show that, an improvement of efficiency benefits for the cooperation fundamentally. Furthermore, comparing with the result on BA scale-free networks, the evolution of cooperation performs great stability on WS small-world networks against the change of market efficiency. As the network of investment in real world may possess both of the properties of WS small-world networks and BA scale-free networks, the findings may be helpful in understanding and controlling the behaviors on the network of investment.

Physica A 506 (2018) 424–432

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Effects of investors' power correlations in the power-based game on networks

Hedong Xu [a], Cunzhi Tian [a], Wenxing Ye [b], Suohai Fan [b,*]

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HIGHLIGHTS

- Effects of investors' power correlations in the power-based game on networks are studied.
- The power correlations is measured by the assortativity coefficient r.
- The expected payoff of a cooperator is more than that of a defector as the level of assotativity is high enough.
- An increment of assortativity coefficient raises the average payoffs of cooperators and boosts cooperations.
- As the market efficiency α swings, the density of cooperators will be higher and more stable on the network with the larger r.

Contents

Background

- 市场中的投资者相互联系。

  Investors are connected as networks in the market.
- 现实中的市场并非完全有效的。

  Markets are not perfect efficient.
- 投资者之间存在能力的差异，如信息获取能力、市场影响力等。

  There exists difference among investors, such as information acquisitions powers.
- 投资者之间影响力的差异，使得投资者在市场中表现产生分歧。

  Differences of powers influence investors' behaviors in the market.

Problem

在上面所描述的市场环境中，投资者会采用怎么样的策略？

What strategies investors will adopt for maximizing payoffs in such markets (Not enough efficiency, different powers, investors are connected)?

Power-based game

假设投资者可以选择合作或者背叛，收益矩阵如下。

Assuming investors will adopt cooperation (C) or defection (D), the payoff matrix is as follow.

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1 & 0 \\ (d_i/d_j)^{\alpha} & 0 \end{pmatrix} \end{array}$$

- d_i, d_j 分别表示投资者 i 和 j 的度。

  d_i, d_j are degrees of investor i and j respectively.

- α 衡量市场有效程度，规定 $\alpha \in [0, 1]$.

  α measures the efficiency of market that varies in $[0, 1]$.

Power-based game

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1 & 0 \\ (d_i/d_j)^{\alpha} & 0 \end{pmatrix} \end{array}$$

- 度值代表投资者的信息获取能力和影响力。

  Degrees represent powers of investors.

- 度值较大的投资者，如机构投资者，有一定的市场影响力；度值较小的投资者，如散户，缺乏市场影响力。

  Investors of greater degrees such as institutional investors own greater powers, but investors of lower degrees such as retail investors own less powers.

Power-based game

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1 & 0 \\ (d_i/d_j)^{\alpha} & 0 \end{pmatrix} \end{array}$$

- 双方合作，资源共享，获取收益一致。由于市场影响力，度值较大投资者背叛度值较小合作者获利更多；相反，度值较小投资者背叛度值较大合作者损失惨重。

  Investors capture the same payoffs if cooperating with each other. Greater investors can get more when he defects smaller investors, but smaller investors will suffer losses if he defects greater investors.

- 影响力的获利程度取决于市场有效程度，即市场能否及时消化信息。

  The profitability of powers depends on the efficiency of the market.

Power-based game on networks

Rules:

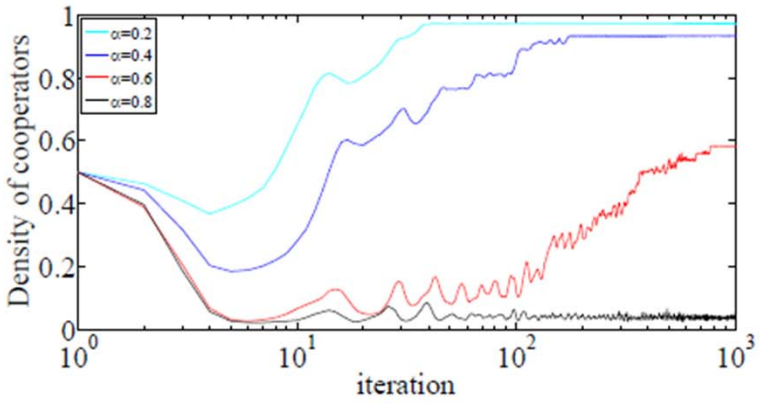
- 投资者在网络中进行多轮博弈。

  Investors play power-based games dynamically on networks.
- 投资者的收益等于它与所有邻居博弈的总和。

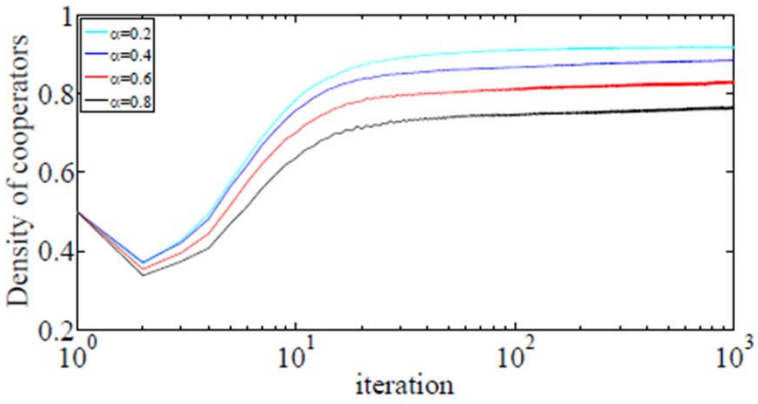
  Investors gain their total payoffs acquired from games with their nearest neighbors traditionally.
- 策略更新规则：费米法则。

  Strategy updating：Fermi function.

Evolutionary processes on networks



(a) BA scale-free networks



(b) WS small-world networks

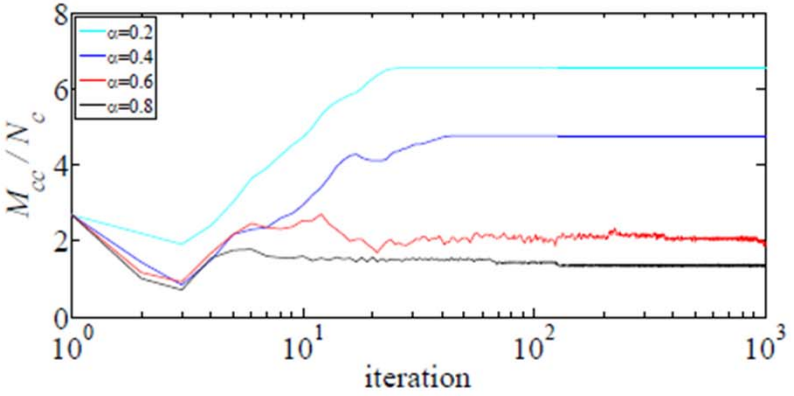
- 市场越有效（$\alpha\downarrow$），合作比率越高。

  As the market is more efficient ($\alpha\downarrow$), the density of cooperation is higher.

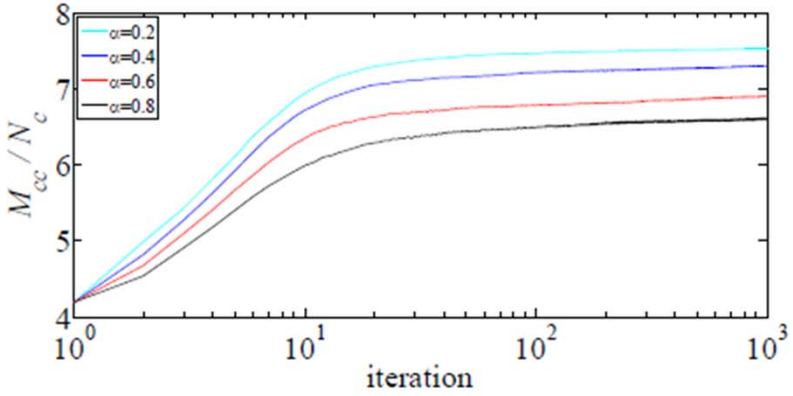
- 随着市场有效程度波动，小世界网络的合作比率更稳定。

  Cooperations performs greater stability on WS small-world networks.

Evolutionary processes on networks



(c) BA scale-free networks

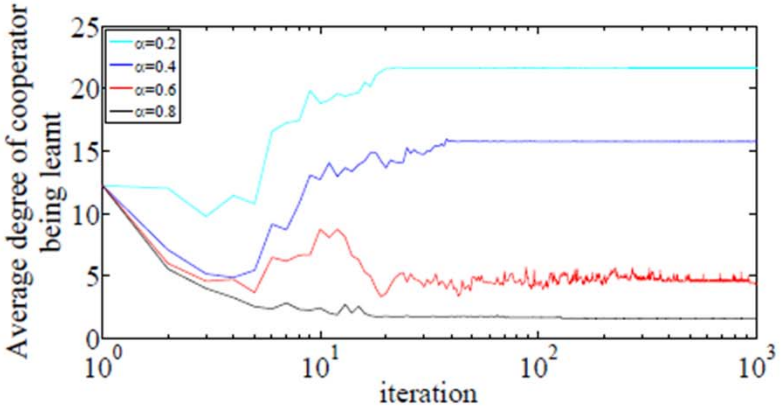


(d) WS small-world networks

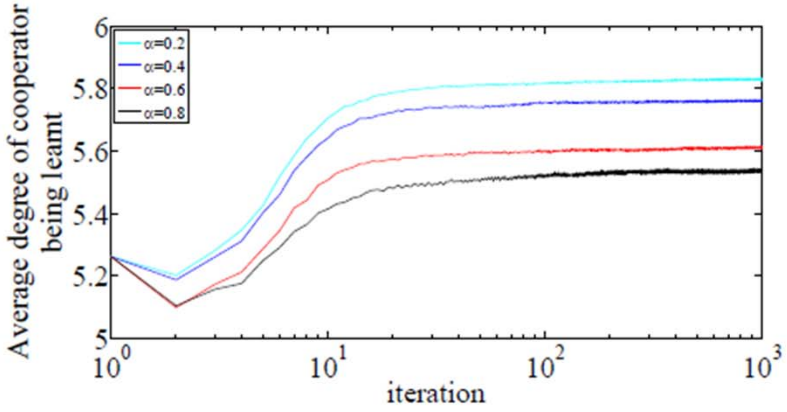
- 市场越有效（$\alpha \downarrow$），合作者聚集程度越高。

  As the market is more efficient ($\alpha \downarrow$), more cooperators are clustering.

Evolutionary processes on networks



(e) BA scale-free networks

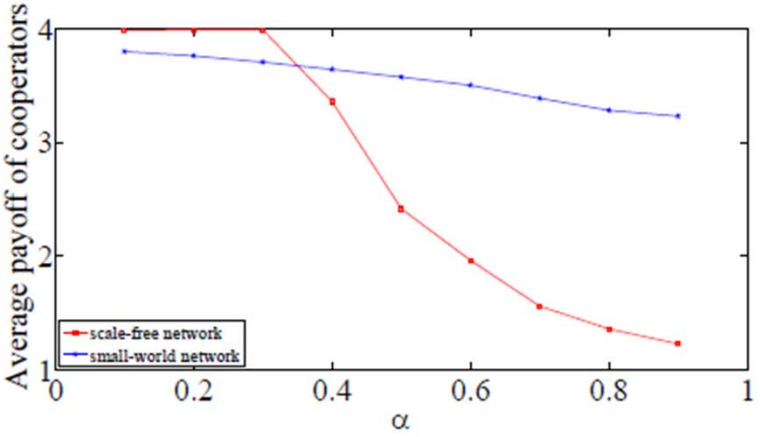


(f) WS small-world networks

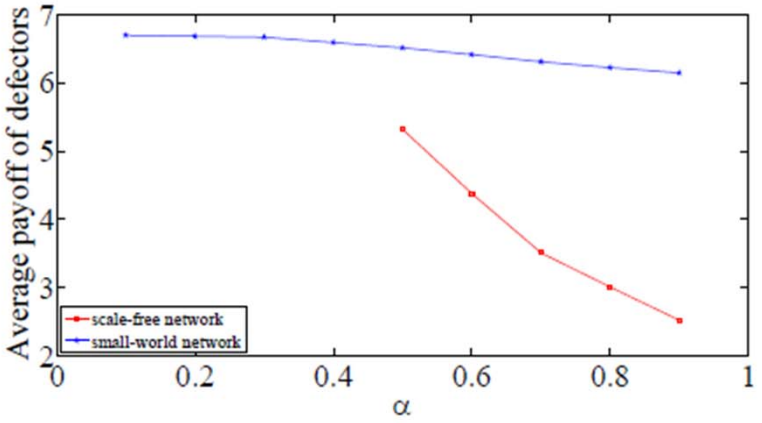
- 市场越有效（$\alpha \downarrow$），合作主要由度值大投资者传播。

  As the market is more efficient ($\alpha \downarrow$), cooperators with higher degree are learnt.

Evolutionary processes on networks



(g)

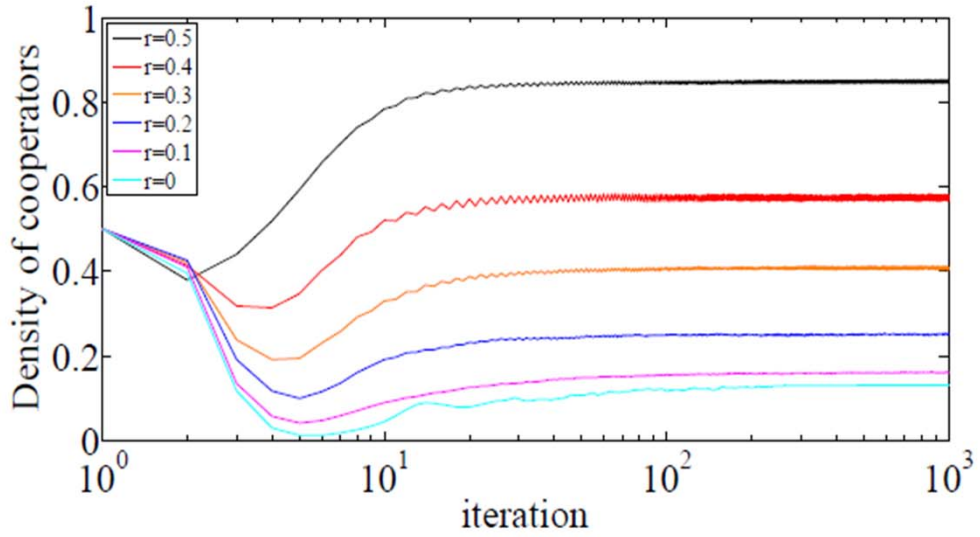


(h)

- 市场有效程度波动，BA 网络上投资者收益波动更大。

  As the market efficiency varies, investors' payoffs are of great volatilities on BA networks.

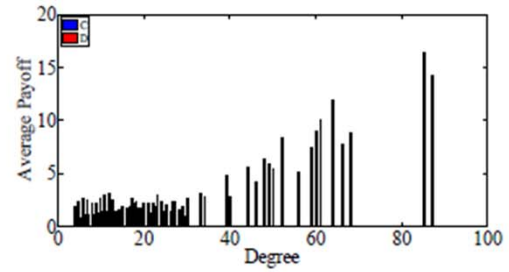
Evolutionary processes on networks



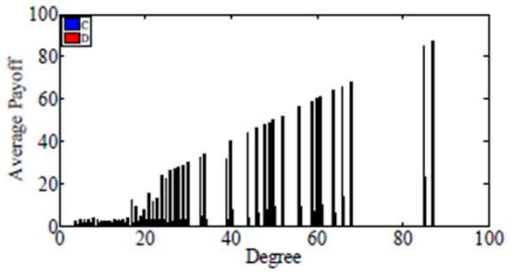
- 投资者同配程度越高（$r\uparrow$），合作比率越高。

  As the assortativity is higher ($r\uparrow$), the density of cooperation is higher.

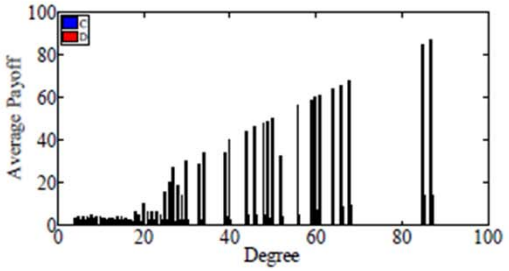
Evolutionary processes on networks



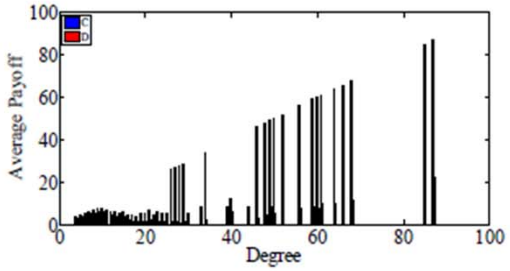
(i) $r = 0.2$



(j) $r = 0.3$



(k) $r = 0.4$

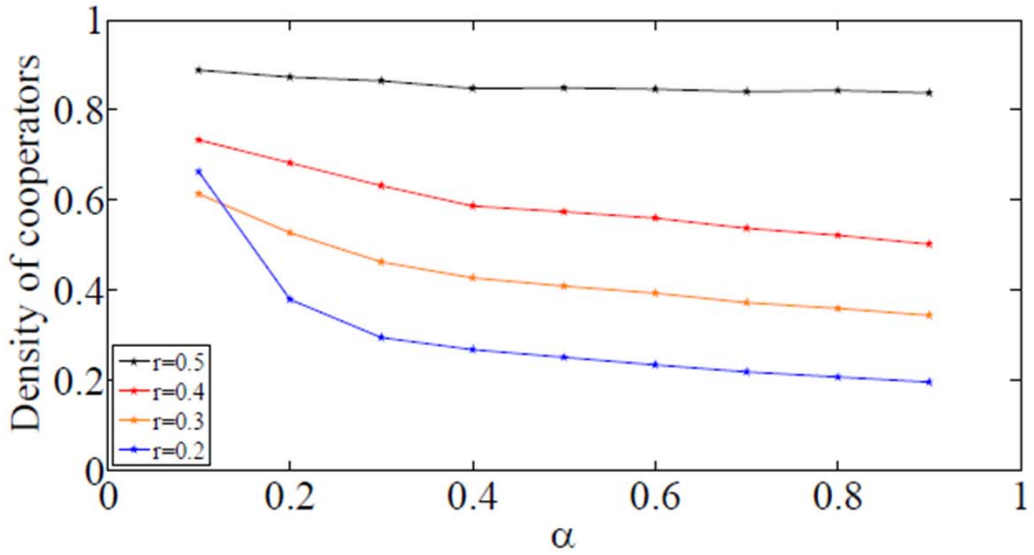


(l) $r = 0.5$

- 投资者同配程度越高（$r\uparrow$），合作收益越高。

  As the assortativity is higher ($r\uparrow$), the payoff of cooperator is higher.

Evolutionary processes on networks



- 投资者同配程度越高（$r\uparrow$），合作比率越稳定。

  As the assortativity is higher ($r\uparrow$), the density of cooperation is of greater stability.

Main results

- 有效率的市场环境和同配的投资者关系，有利于合作的涌现。

  As the efficiency of market ($\alpha \downarrow$) and the assortativity is higher ($r \uparrow$), more cooperators will emerge.

- 小世界和同配的网络结构，使得合作行为更稳定。

  Cooperations are of greater stabilities on small-world networks or assortative networks.

投资者并不“孤独”

- ✓ 投资者之间形成社交网络，“小世界”or“寡头独大”
- ✓ 人脉越多，信息优势越大

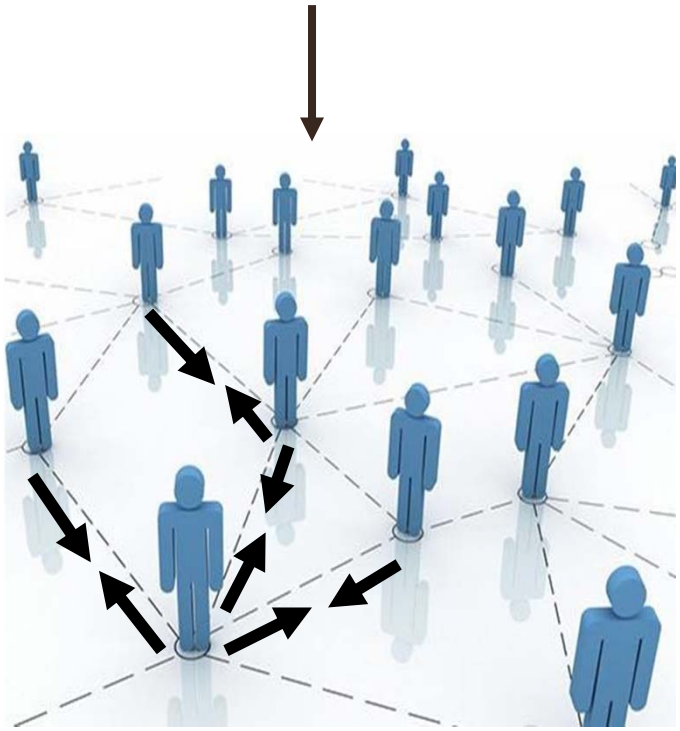
So，在投资者的江湖里，会发生怎么样的故事?

投资者的“心事”

Want to follow…

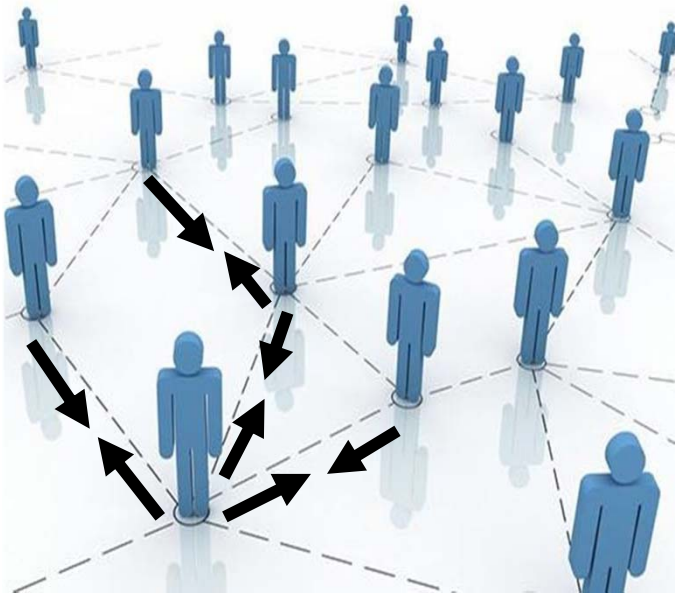
散户 ⟶ 机构投资者

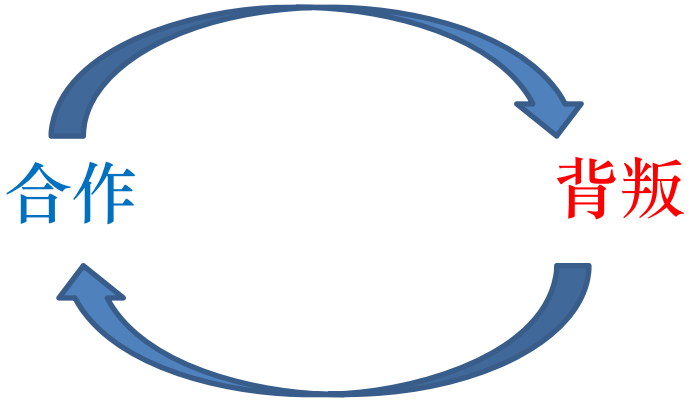
Want to cheat…



每个投资者都不知道对方的角色，
整个投资者的世界最后会合作or背叛?

模拟投资者的世界





随着时间，每个人都在合作和背叛之间纠结与徘徊……

最后的结局

1、市场越成熟，投资者越倾向合作。

2、“寡头独大”投资者社交关系，不利于合作的稳定。

3、投资者分层越明显，投资者越倾向合作，合作行为越稳定。

在投资者的江湖，

尽管每个人都追逐收益，

尽管每个人都在博弈、在猜心事，

只要外部环境良好，

和谐合作总是最好！

谢谢！