Magic Tricks and de Bruijn Sequences

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Magical Mathematics

In Chapters 2 to 4 of *Magical Mathematics*, the authors describe a trick making use of the de Bruijn sequences.



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Binary Number Trick

57 26 58 17 22 48 59 18 23 28 40 54 55 60 29 50 19 24 20 25 30 51 *********** 57 44 26 51 45 58 40 27 -53 3 13 23 33 41 46 59 15 28 10 55 35 45 25 5 15 47 60 11 24 29 42 57 12 25 30 43 56 ***** 32 37 42 47 52 57 42 51 33 38 43 48 53 58 14 34 39 44 49 54 59 46 55 35 40 45 50 55 60 47 58 36 41 46 51 56 * 50 19 30



Why so many cards?

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The Solution - de Bruijn Sequence



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Hamming Code Tricks

- Now we allow the audient to lie at most once.
- As a trade-off, we need to ask more questions.
- To guess an integer chosen from 1 to 2^k, the magician has to ask k + r questions, where r is the smallest integer such that k ≤ 2^r − r − 1.
- If k = 4, the magician has to ask 7 questions. The magic is equivalent to the decoding of the error-correcting Hamming code [7,4,3].

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Hamming Code [7, 4, 3] Trick

			1	3	5		7		2		3	6	7		
			9	11	1	3	15		10	1	1	14	15		
								l							
			4	5	6		7		8		9	10	11		
			12	13	14	£ :	15		12	1	3	14	15		
				_	_			l							
1	2	5	6			1	3	4		6		2	3	4	5
8	11	12	15			8	10	13	3 1	5		8	9	14	15

The Solution to Hamming Code [7, 4, 3] Trick



The New Problem

- There are 2^{2^{k-1}-k} different de Bruijn sequences of order k. For k = 5, there are 2048 different de Bruijn sequences.
- Not every de Bruijn sequence works for the Hamming code trick.

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Our Results

Theorem (Guo-Y.-Hsu, 2019)

For the Hamming code trick with 2^k numbers, there is a de Bruijn sequence of order k which can be used for the one-question-card design.

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Thank you!

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