

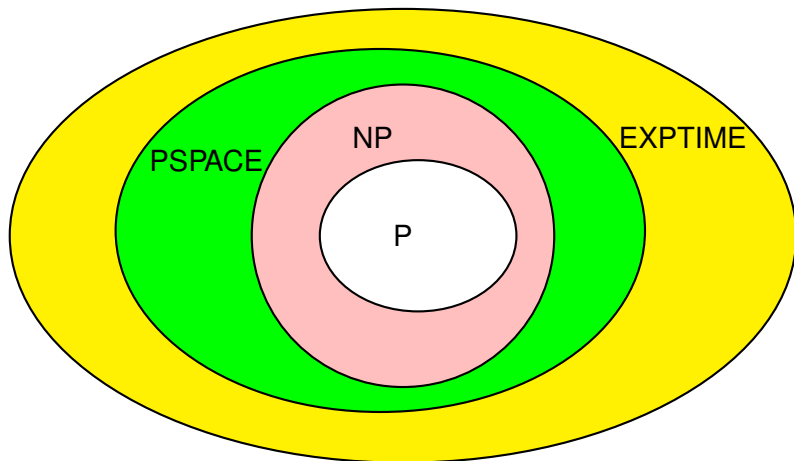
# The Non-deterministic Constraint Logic (NCL) and Its Applications in Computational Complexity

Chao Yang  
杨超

School of Mathematics  
Sun Yat-Sen University, Guangzhou, China

Young Graph Theorists Workshop, 2017  
Shanghai Center for Mathematical Sciences

# Computational Complexity



# The Non-deterministic Constraint Logic (NCL) Problem

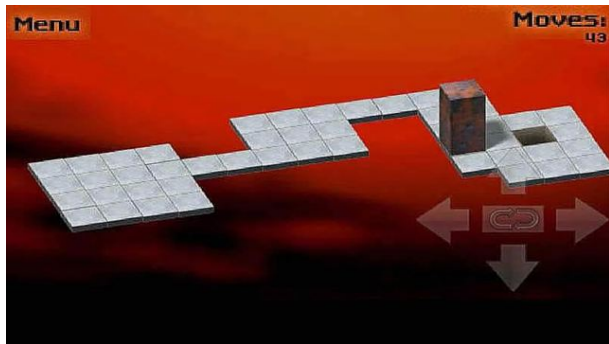
- ▶ A 3-regular digraph.
- ▶ Each edge has weight either 1 or 2.
- ▶ Two kinds of vertices: AND, OR.
- ▶ Constraint: the sum of the weights of the incoming edges is at least 2.
- ▶ Configuration: a valid orientation of the digraph.
- ▶ Question: Given an initial configuration and a target configuration, can we reach the target configuration from the initial configuration by reversing the directions of some edges one by one, under the constraint that all intermediate configurations are valid?

# Planar NCL Problem

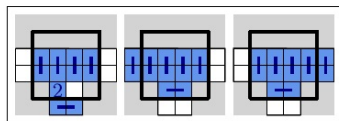
Theorem (Hearn and Demaine, 2005)

*The planar NCL problem is PSPACE-complete.*

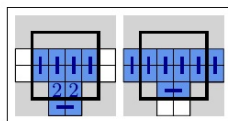
# Application: Rolling Block Puzzle



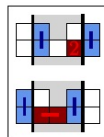
# Rolling Block Puzzle - The Reduction



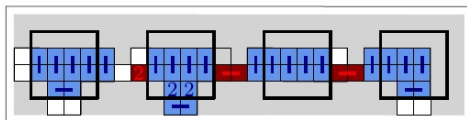
(a) OR-gadget



(b) AND-gadget





(c) inbetween-gadget



(d) Several gadgets connected by the construction in (c)

**Fig. 2** Gadgets for the PSPACE-hardness reduction. A line across two blocks indicates a lying  $2 \times 1 \times 1$  block. A '2' indicates a standing  $2 \times 1 \times 1$  block.

# References

-  R. A. Hearn and E. D. Demaine, PSPACE-completeness of sliding-block puzzles and other problems through the nondeterministic constraint logic model of computation, *Theoretical Computer Science* **343** (2005) 72-96.
-  K. Buchin and M. Buchin, Rolling Block Mazes are PSPACE-complete, *Journal of Information Processing* **20** (2012) 719-722.

Thank you!