Uniform methods and results on the spectral property of hamiltonian problem

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(Jointed work with **Hong-Jian Lai, K.C. Das, Yang Wu**)

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Background

- Let $\Theta(G,\alpha)$ be the largest eigenvalue of $A(G)+\alpha D(G)$, where $\alpha \geq 0$. Let $\rho(G)$ and $\mu(G)$ be the spectral radius and signless Laplacian spectral radius of G, respectively. Let $\delta(G)$ be the minimum degree of G. ([Q. Liu, Y. Hong, X. Gu, H.-J. Lai, LAA, 2014], [M. Liu, B. Liu, Extremal Theory of Graph Spectrum, 2018])

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- A graph G is said to be Hamiltonian (traceable) if it contains a Hamiltonian cycle (path), and a graph G is called Hamilton-connected if any two vertices are connected by a Hamiltonian path in G.

- For any nonnegative integer q, a graph G with $n \geq 3$ vertices is called *q-traceable* if any removal of q vertices to G results to a traceable graph, while a graph G with $n \geq 3$ vertices is called *q-hamiltonian* if any removal of q vertices to G results to a Hamiltonian graph.

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- Recently, the (signless Laplacian) spectral property of Hamiltonian (traceable, Hamilton-connected) graphs received much attention.

- For a nonnegative integer k, the *k-closure*, denoted by $C_k(G)$, of a graph G is the graph obtained from G by recursively joining pairs of nonadjacent vertices whose degree sum is at least k until no such pair remains. It is easy to see that $G \subseteq C_k(G)$.

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Proposition [J.A. Bondy, V. Chvátal, *Discrete Math.*, 1976.]:
(i) "G is q-connected" iff "$C_{n+q-2}(G)$ is q-connected".
(ii) "G is q-edge-connected" iff "$C_{n+q-2}(G)$ is q-edge-connected".
(iii) "G is q-Hamiltonian" iff "$C_{n+q}(G)$ is q-Hamiltonian".
(iv) "G is q-edge-Hamiltonian" iff "$C_{n+q}(G)$ is q-edge-Hamiltonian".
(v) "G is q-path-coverable" iff "$C_{n-q}(G)$ is q-path-coverable"
(vi) "G is q-traceable" iff "$C_{n+q-1}(G)$ is q-traceable" (Liu, Das, Lai).

Spectral results of 0-traceable graphs

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- Spectral results of 0-traceable graphs on n vertices, and $k \geq 0$:

$$N_n^{k,s} \cong K_k \vee \Big(K_{n+s-2k-1} \cup (k+1-s)K_1\Big), \quad \text{and}$$

$$M_n^{k,s} \cong \begin{cases} K_s \vee \big(K_{n-k-1} \cup K_{k+1-s}\big) & \text{for } s > 0, \\ K_{n+s-k-1} \cup H_0, \text{ where } H_0 \text{ is a } k\text{-regular graph} & \text{for } s \leq 0. \end{cases}$$

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$\delta(G)$	$\rho(G)$	Paper	Exception
≥ 0	$\geq n-2$	M. Fiedler, V. Nikiforov, *Linear Algebra Appl.*, 2010.	$N_n^{0,0}$
$\geq k$	$\geq \rho\left(N_n^{k,0}\right)$	B. Li, B. Ning, *Linear Multilinear Algebra*, 2016.	$N_n^{k,0}$
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- $\rho\left(N_n^{k,0}\right) > n-k-2$ (V. Nikiforov, *Czechoslovak Math. J.* 2016).

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≥ 3	$\geq \sqrt{n^2 - 6n + 19}$	Q. Zhou, L. Wang, *Lin. Multil.Algebra*, 2017.	
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- Y. Li, Y. Liu, X. Peng, *Linear Multilinear Algebra*, 2017. showed that: If $\delta(G) \geq k \geq 1$ and $\mu(G) \geq 2(n-k-1)$, then G is Hamiltonian unless G is either obtained from $N_n^{k,1}$ deleting at most $\left\lfloor \frac{1}{4}k^2 \right\rfloor$ edges or G is obtained from $M_n^{k,1}$ by deleting at most $\left\lfloor \frac{1}{4}k \right\rfloor$ edges.

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- Recently, Q. Zhou, L. Wang, Y. Lu, arXiv:1711.11257v1. showed that: If $\delta(G) \geq k \geq 2$ and $\mu(G) \geq 2(n-k)$, then G is Hamilton-connected unless G is either obtained from $N_n^{k,2}$ deleting at most $\frac{1}{4}k(k-1)$ edges or G is obtained from $M_n^{k,2}$ by deleting at most $\frac{1}{2}(k-1)$ edges.

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- (i) For a graph G with $n \geq q+1 \geq 2$ vertices and minimum degree at least one, if $\rho(G) \geq \sqrt{n(n-4)+2q+1}$, then G is q-connected.
  (ii) For a graph G with $n \geq q+1 \geq 3$ vertices and minimum degree at least one, if $\rho(G) \geq \sqrt{n(n-q-5)+2q(q+1)+5}$, then G is q-edge-connected.
  (iii) For a graph G with $n \geq 5q+6 \geq 16$ vertices and minimum degree at least one, if $\rho(G) \geq \sqrt{(n-q-2)^2+q+1}$, then G is q-path-coverable.

Our results

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Definition: M. Liu, H.-J. Lai, C. Das, Paper submitted

Suppose that $s_0 = \max\{0, s\} \leq p \leq k \leq \frac{1}{2}(n+s-2)$.

If $s_0 = p \leq k$, then let $\mathbb{G}_n(p,s,k) = \left\{M_n^{k,s}\right\}$.

If $s_0 + 1 \leq p \leq k$, let $\mathbb{G}_n(p,s,k) = \Big\{G : G \cong K_p \vee \left(K_{n+s-k-1-p} \cup H_0\right)$, where H_0 is a $(k-p)$-regular graph with $k+1-s$ vertices$\Big\}$.

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Definition: M. Liu, H.-J. Lai, C. Das, Paper submitted

A graph G with n vertices and minimum degree $\delta(G) \geq k$ is called an *s-suitable graph* if G satisfies the following conditions: (i) K_n is s-suitable, (ii) G is s-suitable if and only if $C_{n+s-1}(G)$ is s-suitable, and (iii) If $s_0 \leq p \leq k \leq \frac{1}{2}(n+s-2)$ and $G \in \mathbb{G}(p,s,k)$, then G is s-suitable if and only if $p \in \{s_0+1, s_0+2, \ldots, k-1\}$, where $s_0 = \max\{0, s\}$.

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- Let

$$\Theta_0 = \alpha\left(\frac{2\varepsilon_0}{n-1} + n - 2\right) + \frac{1}{2}(1-\alpha)\left(k - 1 + \sqrt{(k+1)^2 + 8\varepsilon_0 - 4nk}\right)$$

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Theorem: M. Liu, H.-J. Lai, C. Das, Paper submitted

Let s and k be two integers and let G be a graph with $n \geq 6k + 10 - 5s$ vertices and minimum degree $\delta(G) \geq k \geq \max\{1, s\}$. If $\Theta(G, \alpha) > \Theta_0$ and $0 \leq \alpha \leq 1$, then either $C_{n+s-1}(G) \cong K_n$ or $C_{n+s-1}(G) \in \mathbb{G}_n(p, s, k)$ holds for some integer p, where $\{0, s\} \leq p \leq k$.

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If $\delta(G) \geq k \geq \max\{1, s\}$, $\Theta(G, \alpha) \geq \Theta\left(N_{n,0}^{k,s}, \alpha\right)$ and $0 \leq \alpha \leq 1$, then either $C_{n+s-1}(G) \cong K_n$ or $G \in \left\{\mathbb{G}_n(p, s, k) : \text{where } \{0, s\} \leq p \leq k\right\}$ unless $G \cong N_{n,0}^{k,s}$.

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- $\rho\left(N_{n,0}^{k,s}\right) < n + s - k - 2$ and

$$\mu\left(N_{n,0}^{k,s}\right) < \frac{2n^2 - 2(k+3-s)n + (k-s+1)(3k-s+2)}{n-1}.$$

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(i) If $0 \leq q \leq p \leq k \leq \frac{1}{2}(n+q-2)$, and $G \in \mathbb{G}_n(p,q,k)$, then G is q-traceable if and only if $p \in \{q+1, q+2, \ldots, k-1\}$.

(ii) If $1 \leq q+1 \leq p \leq k \leq \frac{1}{2}(n+q-1)$ and $G \in \mathbb{G}_n(p,q+1,k)$, then G is q-hamiltonian if and only if $p \in \{q+2, q+3, \ldots, k-1\}$.

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- Let $\mathbb{G}_n$ be the class of graphs with n vertices and q be a nonnegative integer. If G has the property P if and only if $C_q(G)$ has property P for each $G \in \mathbb{G}_n$, then the property P is said to be *q-stable*.

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(ii) If $1 \leq q+1 \leq p \leq k \leq \frac{1}{2}(n+q-1)$ and $G \in \mathbb{G}_n(p,q+1,k)$, then G is q-hamiltonian if and only if $p \in \{q+2, q+3, \ldots, k-1\}$.

- Let $\mathbb{G}_n$ be the class of graphs with n vertices and q be a nonnegative integer. If G has the property P if and only if $C_q(G)$ has property P for each $G \in \mathbb{G}_n$, then the property P is said to be *q-stable*.
- In [J.A. Bondy, V. Chvátal, *Discrete Math.*1976] and [H. Broersma, Z. Ryjáček, I. Schiermeyer, *Graphs Combin.*2000], many q-stable properties had been given. Thus, we can apply our results to give a new spectral property to these q-stable properties.

(Signless Laplacian) spectral results of complement graphs on Hamiltonian problem

Definition: M. Liu, Y. Wu, H.-J. Lai, Paper submitted

Let $\mathbb{B}_{n,k,s,r} = \Big\{\overline{G_1} \vee G_2 : G_1 = (U, V)$ is a connected $(k-s, n-k-1)$-semi-regular bipartite with $n-s-1+r$ vertices and G_2 is a spanning subgraph of K_{s+1-r}, where $0 \leq r \leq s+1$ and $r \neq 1\Big\}$.

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If $n = 2k+1-s$ and $s \leq 1$, then let $\mathbb{H}_{n,k,s,r} = \Big\{G_1 \vee G_2 : G_1$ is a r-regular graph with $n-k+r$ vertices and G_2 is a spanning subgraph of K_{k-r}, where $0 \leq r \leq k\Big\}$.

(Signless Laplacian) spectral results of complement graphs on Hamiltonian problem

Definition: M. Liu, Y. Wu, H.-J. Lai, Paper submitted

If $s \geq -1$, then we denote by $\mathbb{C}_{n,s,r} = \Big\{\overline{G_1} \vee G_2 : G_1$ is a connected $(p, n-s-1-p)$-semi-regular bipartite with $n-s-1+r$ vertices and G_2 is a spanning subgraph of K_{s+1-r}, where $0 \leq r \leq s+1$, $r \neq 1$ and $1 \leq p \leq \frac{n-s-1}{2}\Big\}$.

Definition: M. Liu, Y. Wu, H.-J. Lai, Paper submitted

Let $\mathbb{D}_{n,s,r} = \Big\{\overline{G_1} \vee G_2 : G_1$ is a connected $\frac{n-s-1}{2}$-regular graph with $n-r$ vertices and G_2 is a spanning subgraph of K_r with $\mu(\overline{G_2}) \leq n-s-1$, where $0 \leq r \leq \frac{n+s-1}{2}\Big\}$.

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- The spectral property of complement graphs on traceable problem. Suppose that G is a graph with n vertices and $k \geq 0$.

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$\geq k$	$\leq \rho\left(\overline{M_n^{k,0}}\right)$	B. Li, B. Ning, *LMA*, 2016.	$G \in \mathbb{B}_{n,k,-1,r}$ or $G \in \mathbb{H}_{n,k,-1,0}$

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- Later, M.-Z. Chen, X.-D. Zhang, *J. Comb. Optim.*, 2018 and G. Yu, Y. Fang, Y. Fan, G. Cai, arXiv:1705.01683v2. independently proved that:

Theorem

For any graph with $n \geq 2k$ vertices and minimum degree $\delta(G) \geq k \geq 2$, if $\rho(\overline{G}) \leq \sqrt{(k-1)(n-k-1)}$, then G is Hamilton-connected unless $G \in \mathbb{B}_{n,k,1,0}$ or $G \in \mathbb{H}_{n,k,1,r}$.

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For any graph with $n \geq 2k+1$ vertices and minimum degree $\delta(G) \geq k \geq q+1 \geq 1$, if $\rho(\overline{G}) \leq \sqrt{(k-q)(n-k-1)}$, then G is q-Hamiltonian and q-edge-Hamiltonian unless $G \in \mathbb{B}_{n,k,q,r}$ or $G \in \mathbb{H}_{n,k,q,r}$.

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If minimum degree $\delta(G) \geq k \geq \min\{q-1, 1\}$, $q \geq 1$ and $\rho(\overline{G}) \leq \sqrt{(k-q+2)(n-k-1)}$, then G is q-connected and q-edge-connected unless $G \in \mathbb{B}_{n,k,q-2,r}$ or $G \in \mathbb{H}_{n,k,q-2,r}$.

Theorem

If $\delta(G) \geq k \geq 1$, $q \geq 1$ and $\rho(\overline{G}) \leq \sqrt{(k+q)(n-k-1)}$, then G is q-path-coverable unless $G \in \mathbb{H}_{n,k,-q,r}$ or $G \cong K_{k+1} \cup K_{n-k-1}$ when $q = 1$.

Signless Laplacian spectral results of complement graphs on q-traceable problem

Theorem B. Zhou, *Linear Algebra Appl.*, 2010.

If $\mu(\overline{G}) \leq n$, then G is traceable unless $G \in \mathbb{C}_{n,-1,r}$ or $G \in \mathbb{D}_{n,-1,r}$.

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Main results

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Let k and s be two integers and let G be a graph with $n \geq \max\{2k, 2k+1-s\}$ vertices and minimum degree $\delta(G) \geq k \geq \max\{s, 1\}$. If $\rho(\overline{G}) \leq \sqrt{(k-s)(n-k-1)}$ and $C_{n+s}(G) \ncong K_n$, then either $G \in \mathbb{B}_{n,k,s,r}$ or $G \in \mathbb{H}_{n,k,s,r}$.

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Let s be an integer and let G be a graph with $n \geq 3s+2$ vertices. If $\mu(\overline{G}) \leq n-s-1$ and $C_{n+s}(G) \ncong K_n$, then either $G \in \mathbb{C}_{n,s,r}$ or $G \in \mathbb{D}_{n,s,r}$.

(Signless Laplacian) spectral results of bipartite graphs on Hamiltonian problem

- Recently, the (signless Laplacian) spectral property of Hamiltonian (traceable, Hamilton-connected) bipartite graphs received much attention.

Definition.

A bipartite graph $G = (U, V)$ is called (almost) balanced if $|U| = |V|$ ($|U| = |V| + 1$), where U and V are the two partite sets of G.

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- For the spectral property of traceable bipartite graphs, Liu et al. proved that:

Theorem [R. Liu, W. Shiu, J. Xue, *LAA*,2015].

For a balanced bipartite graph G with $2n \geq 6$ vertices and minimum degree $\delta(G) \geq 1$, if $\rho(G) \geq \sqrt{n^2 - 2n + 3}$, then G is traceable.

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- For the spectral property of Hamiltonian bipartite graphs, Liu et al. proved that:

Theorem [R. Liu, W. Shiu, J. Xue, *LAA*,2015].

For a balanced bipartite graph G with $2n \geq 8$ vertices and minimum degree $\delta(G) \geq 2$, if $\rho(G) \geq \sqrt{n^2 - 2n + 4}$, then G is Hamiltonian unless $G \cong B_{2,n-2;n-2,2}$.

(Signless Laplacian) spectral results of balanced bipartite graphs on Hamiltonian problem

- One year later, Li et al. generalized this result to:

Theorem [B. Li, B. Ning, *LMA*, 2016].

For a balanced bipartite graph G with $2n \geq 2(k+1)^2$ vertices and minimum degree $\delta(G) \geq k \geq 1$, if either $\mu(G) \geq \mu(Z_{n-k,k})$ or $\rho(G) \geq \rho(Z_{n-k,k})$, then G is Hamiltonian unless $G \cong Z_{n-k,k}$.

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- Very recently, Ge et al. improved the former result to:

Theorem [J. Ge, B. Ning, arXiv:1606.08530v3, 2016].

For a balanced bipartite graph G with $2n$ vertices and minimum degree $\delta(G) \geq k \geq 1$, if $n \geq k^3 + 2k + 4$ and $\rho(G) \geq \sqrt{n(n-k)}$, then G is Hamiltonian unless $G \cong Z_{n-k,k}$.

Main results

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Definition. J.A. Bondy, V. Chvátal, *Discrete Math.*1976

Let k be a non-negative integer. The *bipartite closure graph* $B_k(G)$ of a bipartite graph G, is the graph obtained from G by recursively joining pairs of non-adjacent vertices in different vertex parts whose degree sum is at least k until no such pair remains.

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Definition. M. Liu, Y. Wu, H.-J. Lai, Paper submitted

Let $G = (U, V)$ be a bipartite graph and let S be any vertex set of G with $2q$ vertices such that $|S \cap U| = |S \cap V| = q$. For any nonnegative integer q, G is called (q, q)-*traceable* if the removal of S from G results a traceable graph, while G is called (q, q)-*Hamiltonian* if the removal of S from G results a Hamiltonian graph.

Main results

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Proposition. M. Liu, Y. Wu, H.-J. Lai, Paper submitted

Let G be a balanced bipartite graph with $2n$ vertices. If q is a nonnegative integer, then G is (q,q)-traceable if and only if $B_{n+q}(G)$ is (q,q)-traceable, and G is (q,q)-Hamiltonian if and only if $B_{n+q+1}(G)$ is (q,q)-Hamiltonian.

Theorem. M. Liu, Y. Wu, H.-J. Lai, Paper submitted

Let s and k be two non-negative integers and let G be a balanced bipartite graph with $2n \geq 8k(k+1)$ vertices and minimum degree $\delta(G) \geq k \geq \max\{s,1\}$. If either $\rho(G) \geq \rho\left(F^0_{n,k,s}\right)$ or $\mu(G) \geq \mu\left(F^0_{n,k,s}\right)$, then either $B_{n+s}(G) \cong K_{n,n}$ or $G \in \left\{F_{n,k,s},\ F^0_{n,k,s}\right\}$.

- If $n \geq \frac{1}{2}\left(k^2+4\right)(k+1)$ and $k \geq \max\{s,1\}$, then $\rho\left(F^0_{n,k,s}\right) < \sqrt{n(n+s-k-1)}$.

(Signless Laplacian) spectral results of almost balanced bipartite graphs on Hamiltonian problem

- Liu et al. firstly proved that:

Theorem [R. Liu, W. Shiu, J. Xue, *LAA*, 2015.

For an almost balanced bipartite graph $G[U, V]$ with $2n-1$ vertices and $\delta(G) \geq 1$, if $\rho(G) \geq \sqrt{n^2 - 3n + 4}$, then G is traceable unless $G \in \left\{B_{2,n-2;n-3,2},\ B_{n-2,2;1,n-2}\right\}$.

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- Recently, Yu et al. extended Liu et al.'s result to:

Theorem [G. Yu, Y. Fang, Y. Fan, G. Cai, arXiv.].

For an almost balanced bipartite graph G with $2n-1$ vertices and $\delta(G) \geq k \geq 1$, if either $\mu(G) > 2n-k-2+\frac{(k+1)^2}{n}$ or $\rho(G) > \sqrt{n(n-k-1)}$, then G is traceable unless $G \cong Z_{n-k-1,k}$.

Main results

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- Hereafter, denote by

$$\Omega(\alpha) = \alpha\left(2n - k - 2 + \frac{(k+1)^2}{n}\right) + (1-\alpha)\sqrt{k(k+2-n) + (n-1)^2}.$$

Proposition. M. Liu, Y. Wu, H.-J. Lai, Paper submitted

Let G be an almost balanced bipartite graph with $2n-1$ vertices and minimum degree $\delta(G) \geq k \geq 1$. If $n \geq 3k+4$, $0 \leq \alpha \leq 1$, and $\Theta(G,\alpha) > \Omega(\alpha)$, then G is traceable unless $G \subseteq Z_{n-k-1,k}$.

Theorem. M. Liu, Y. Wu, H.-J. Lai, Paper submitted

Let G be an almost balanced bipartite graph with $2n-1$ vertices and minimum degree $\delta(G) \geq k \geq 1$, where $n \geq 4k(k+1)$. If either $\rho(G) \geq \rho\left(Z^0_{n-k-1,k}\right)$ or $\mu(G) \geq \mu\left(Z^0_{n-k-1,k}\right)$, then G is traceable unless $G \in \left\{Z_{n-k-1,k},\ Z^0_{n-k-1,k}\right\}$.

Thank You

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